Mathematical Analysis - List 1

1. List the elements in each of the following sets:

a) $\left\{x^2+1 : x \in \mathbb{Z} \text{ and } -3 < x \leqslant 2\right\}$; b) $\left\{x : x \in \mathbb{Z} \text{ and } x = 2y,\ y^2 \leqslant 17\right\}$;

c) $\{(x,y,z) : x,y,z \in \mathbb{N} \ \wedge\ x < y < z \ \wedge\ xyz = 16\}$;

d) $\{(x,y,z) : x,y,z \in \mathbb{N} \ \wedge\ x < y < z \ \wedge\ xyz = 16\}^2$.

2. Give conditions which precisely describe the following sets:

a) $[-1,7]$; b) $\{1,2,3,5,8,13,21,\ldots\}$;

c) $\left\{\frac{1}{2}, \frac{2}{3}, \frac{4}{5}, \frac{8}{7}, \frac{16}{11}, \frac{32}{13}, \ldots\right\}$; d) $\{-15,-5,-3,-1,1,3,5,15\}$.

3. Find: $A \cup B$, $A \cap B$, $A \setminus B$, $B \setminus A$,

a) $A = (0,5)$, $B = [0,7]$; b) $A = (-\infty,3)$, $B = (-1,\infty)$;

c) $A = \{1,2\}$, $B = \{1,2,3,4\}$; d) $A = (-\infty,3) \cup \{5\}$, $B = (-1,\infty) \setminus \{5\}$;

e) $A = \{1,2,3\}$, $B = (1,3)$; f) $A = (-\infty,3) \setminus \{2\}$, $B = (-1,\infty) \setminus \{4\}$.

4. What is the relationship between the sets A and B, if:

a) $A \cup B = A$; b) $A \cup B \subset A$; c) $A \setminus B = A$; d) $B \subset A \cap B$;

e) $\mathcal{P}(B) \subset \mathcal{P}(A)$; f) $\mathcal{P}(A) \setminus \mathcal{P}(B) = \emptyset$.

5. For $A \subset \mathbb{R}$ find $A^c = \mathbb{R} \setminus A$ if:

a) $A = (0,4) \setminus \{2\}$; b) $A = (0,1) \cup [2,\infty)$;

c) $A = \{1,2\}$; d) $A = (-\infty,3) \cup \{5\}$.

6. Determine whether each of the following statements is true or false.

a) For all sets A, B, and C, if $A \cap C = B \cap C$ then $A = B$.

b) For all sets A, B, and C, if $A \cup C = B \cup C$ then $A = B$.

7. Let A and B be subsets of a set X. Recall that $A^c = X \setminus A$. Show:

a) If $A \subset B$ then $B^c \subset A^c$.

b) De Morgan's Laws: $(A \cup B)^c = A^c \cap B^c$ and $(A \cap B)^c = A^c \cup B^c$.

8. Find the Cartesian product $A \times B$, if:

a) $A = (0,5)$, $B = [0,7]$; b) $A = (-\infty,3)$, $B = (-1,\infty)$;

c) $A = \{1,2\}$, $B = \{1,2,3,4\}$; d) $A = \{1,2,3\}$, $B = \mathbb{R}$;

e) $A = \mathbb{R}$, $B = \{2,3,4\}$; f) $A = \{1,2,3\}$, $B = \emptyset$.

9. Decide whether the following statements with quantifiers are true or false:

a) $\exists \alpha \in \mathbb{R}\ \cos\alpha = -\sqrt{3}$; b) $\forall x \in \mathbb{R} \quad x^2 + 4x + 3 > 0$;

c) $\forall\, x \in \mathbb{R}\ \ \exists\, y \in \mathbb{R}\ \ x^3 - y^2 = 0$; d) $\exists\, y \in \mathbb{R}\ \ \forall\, x \in \mathbb{R}\ \ x^2 - y^2 = 0$;

e) $\exists\, y \in \mathbb{R}\ \ \forall\, x \in \mathbb{R}\ \ xy + x = 0$.